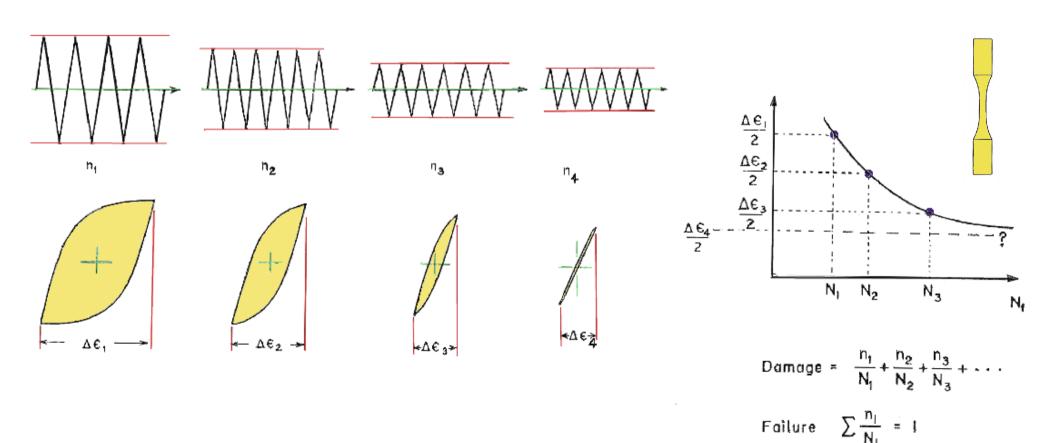


Linear Damage Summation: Palmgren-Miner Rule

Most service load or strain histories are mictures of small and large cycles. By following material memory rules we can pick out closed hysteresis loops, as described previously. Given a fitted fatigue life curve, in this case a strain-life curve, we can estimate what a sequence of different sized closed loops will cause in terms of fatigue crack initiation damage.

For the largest loop shown on the left side below, there are 4 cycles (4 closed loops). If in the fitted curve on the right N₁ is 1000 cycles, then the fraction of life consumed by these 4 loops is damage = 4/1000 The other sized closed loop's effects are evaluated in the same way. When the summation of damage equals 1.0 fatigue crack initiation is assumed.



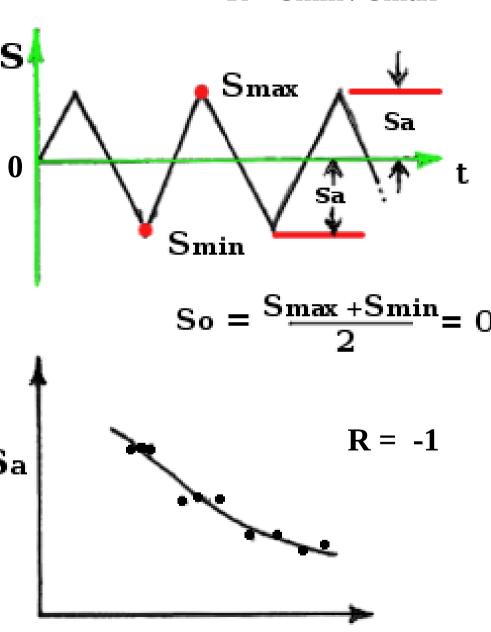
Since our digital fitted curve also includes as stress term, we can perform the same damage summation using stress.

In past testing, and many rotating bending tests, stress was often used to characterize the test or service history. Usually the use of stress was limited to the long life fatigue region, where there is little plasticity in the hot spot stress-strain hysteresis loop. (In rotating bending the hot-spot is at the surface of the round beam)

Researchers that use stress generally define the stress history in terms shown in the upper right figure, and, for damage calculation purposes, use a stress vs. life plot of some sort.

Often their fitted curves show fully reversed data, where the mean stress So = 0 and the stress ration R = -1

R = Smin / Smax

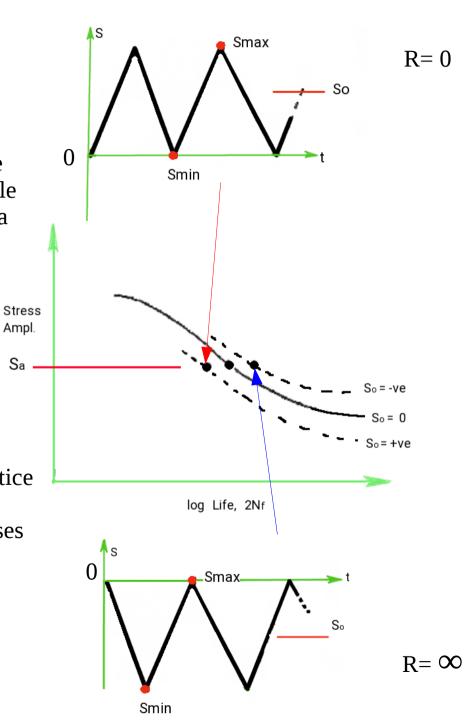


"But what happens when the mean stress is NOT So = 0.2"

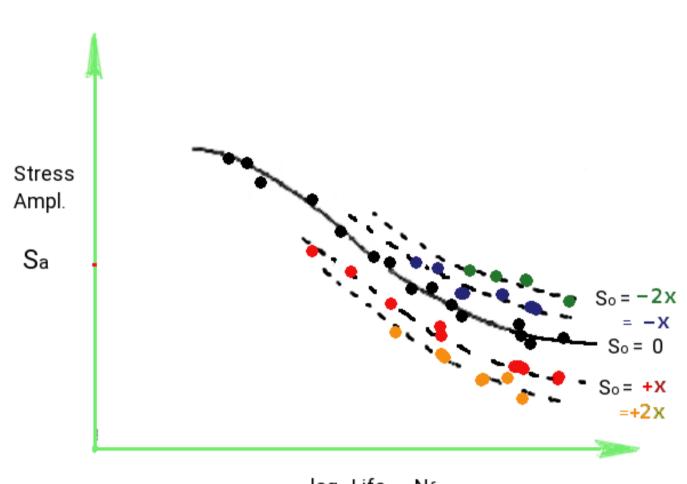
For a the same stress amplitude, a tensile mean stress will shorten fatigue life, while a compressive mean stress test will have a longer fatigue life than tests with zero mean stress.

The effect is generally attributed to tensile stresses stretching the atomic latice and making slip easier, or opening microcracks.; -with compressive stresses having the opposite effects.

Sa



If one wishes to experimentally find the effects of different mean stress values on fatigue life for improved damage summations, one must perform a large number of tests. This quickly becomes uneconomic when done for many materials.



Mean stress correction methods were invented to overcome this problem. There are several available.

log Life, Nf

We will only consider two:

- 1. Smith-Watson-Topper correction parameter (PSWT)
- 2. Morrow mean stress correction method (Mmsc)

The Smith-Watson-Topper mean stress correction method uses the stress and strain information of a resolved closed hysteresis loop to compute the parameter

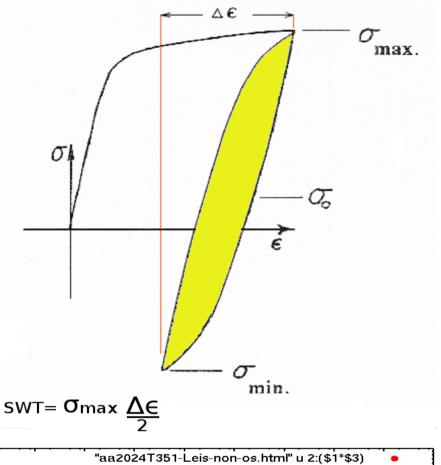
$$P_{SWT} = \sigma_{max} \; \underline{\Delta \epsilon}_{2}$$

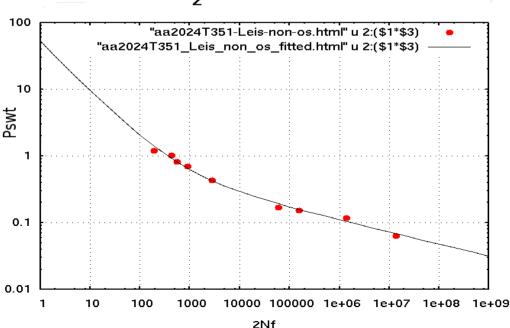
where for the strain amplitude $\Delta\epsilon/2$ defines the size of the loop, and the \mathbf{O}_{max} term, the maximum stress of the loop will increase the size of the product when the mean stress is higher.

When loops are below zero stress however, this parameter does not work.

Fully compressive loops can still initiate fatigue cracks - they just won't propagate.

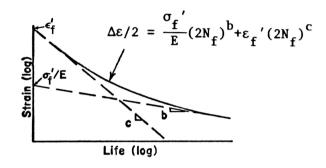
Thus for compressive mean stress cases many engineers use Morrow's correction.

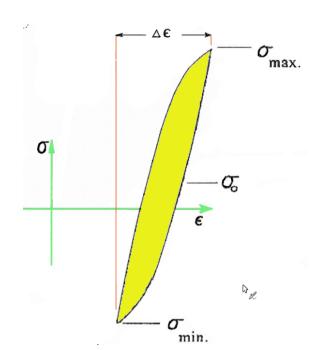




Morrow's mean stress correction

The method uses information from the traditional strain vs life curve fitting process.





Recall that in Chap.7 on database files the traditional fit is a vertical summation of two straight lines.

The σ_f'/E term is the intercept of the Elastic Strain vs Life line.

Morrow moves this line up or down depending on the mean stress.

i.e.: New
$$\sigma_f'/E = (\sigma_f'/E - \sigma_0)$$

Thus for a tensile mean stress the line moves down and shortens life. The new traditional fit strain vs. life curve is then computed and damage is calculated using strain amplitudes.

-but the combined equation is tedious for many repeated calculations...

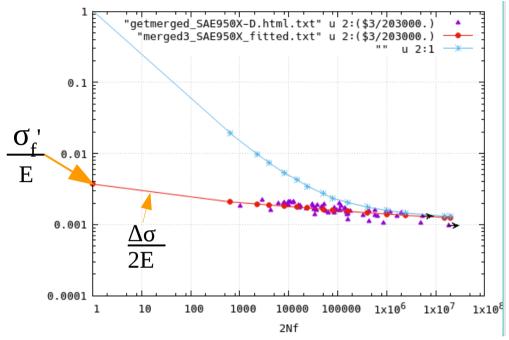
$$\Delta \mathbf{O} = \mathbf{O}_{\text{max}} - \mathbf{O}_{\text{min}}$$

It is easier to just do the correction calculation in stress terms.

Since: Elastic Strain = Stress/Emod we can compute a zero mean stress equivalent amplitude :

$$\frac{\Delta \sigma}{2} \text{ eq } = \Delta \sigma / 2 * \left(\frac{\sigma f'}{\sigma f' - \sigma o} \right)$$

Using this equivalent stress ampl. we can then enter the fitted stress vs life curve to compute the life at this mean stress without solving the traditional strain life equations.



The computer routines we will use in later sections and chapters will predict the life for both of these two mean stress correction methods, and several other less popular methods. It is up to you to decide which method you will apply.

Ref.:

K.N.Smith, P.Watson, T.H.Topper, "A Stress-Strain Function for the Fatigue of Metals," J.of Materials, V5 N4 Dec. 1970 pp.767-778

J.Morrow, Section 3.2 of SAE "Fatigue Design Handbook", 1968.

End of Chapter 8 Part-2.

Return link to Chapter List: https://fde.uwaterloo.ca/FatigueClass/classIndex.html

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Small cycle effects in damage computations
https://fde.uwaterloo.ca/FatigueClass/Chap8Damage/chap8damagePt-2B.pdf