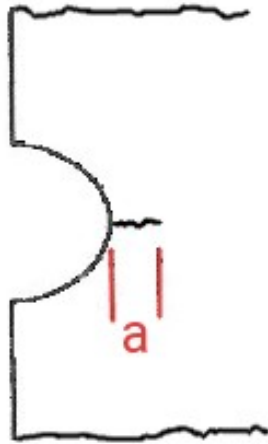


# Crack Propagation Damage

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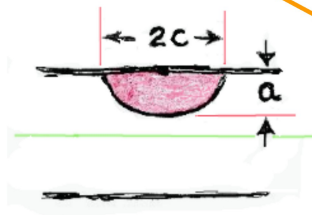
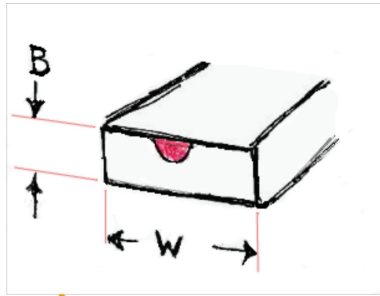


Damage in crack propagation simulation models is assumed to be the increase in fatigue crack length or area as cyclic loading proceeds.

Specific computational techniques vary considerably from industry to industry, but in general the following steps are applied:

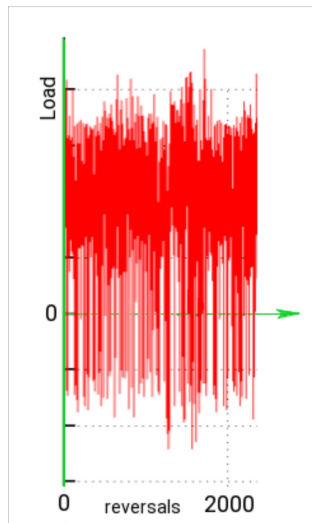
1. Test small specimens under constant amplitude histories (for different R ratios) and generate  $da/dN$  vs.  $\Delta K$  curves.
2. Cycle count the load history using Rainflow or Level Crossing counts.
3. Compute a suitable load to  $\Delta K$  relationship for the expected geometries of the component/structure. These Stress Intensity Factors and finite width corrections are found in handbooks, journals, databases or by using FEA.
4. Compute the progression of crack length "a" etc. by applying the expected cycle sets. Some methods use a material memory model at this point. Many do not. We **will** be using such a model to allow us to simulate each half-cycle as it appears in the load history sequence.

**Estimate  
Initiation  
Crack  
Size**



With crack  $a, c$ , etc.  
compute crack stresses  
and Stress Intensity  $K$   
for the next half cycle

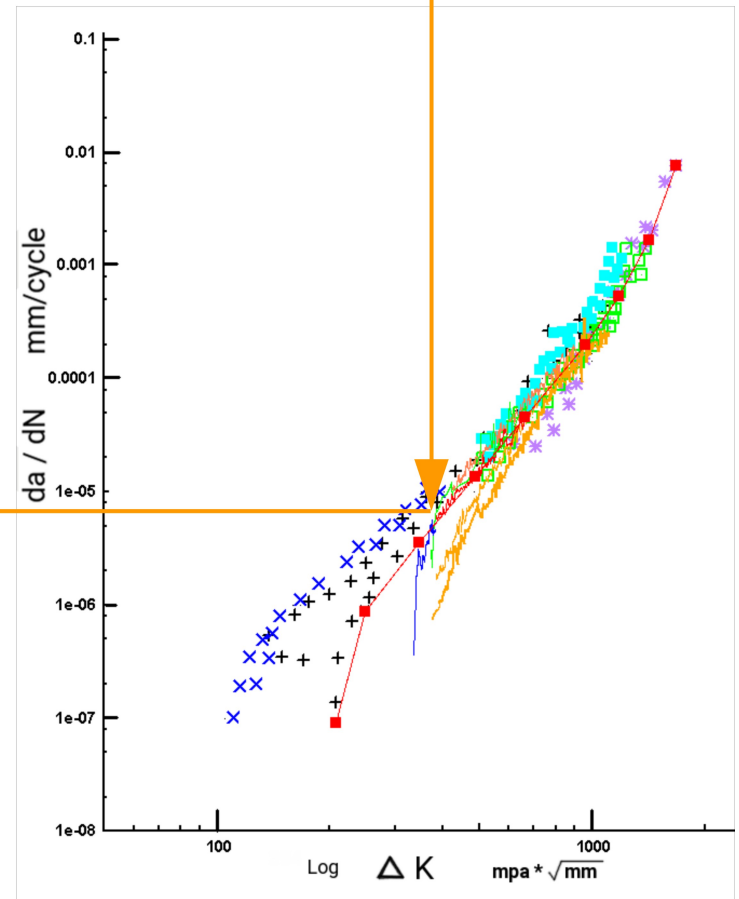
Use a material memory model with  
Stress Intensity  $K$  to compute  
"closed loops", get  $\Delta K$



Get a load, compute  
Bend. Moment and  
Axial Forces on  
section

Update  
and check  
crack lengths  
 $a, c$ , etc.

**Fracture  
?**



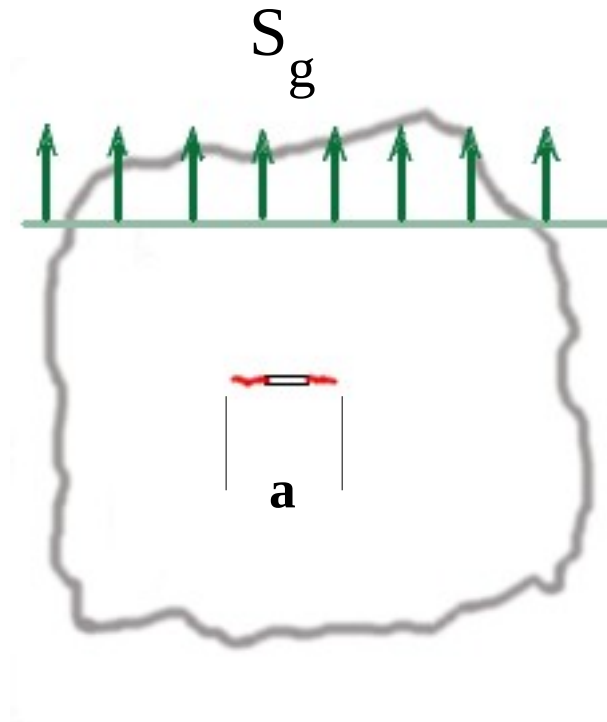
In an infinite sheet or plate with a crack the gross stress  $S_g$  applied to the sheet will cause a "Stress Intensity"  $K$  where:

$$K = S_g \sqrt{\pi a}$$

where  $a$  is the crack length.

Much has been written on this subject and there is little value in repeating it here.

More information: [wikipedia on K](#)



#### Books:

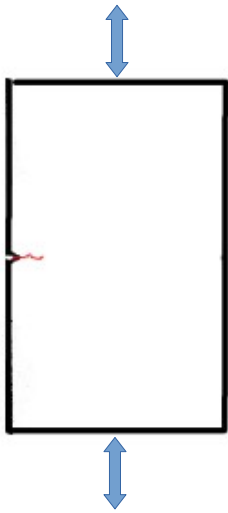
D.P.Rooke, D.J.Cartwright, "Compendium of Stress Intensity Factors," ISBN 0 11 771336 8, Her Majesties Stationary Off., 1976

Y.Murakami, "Stress Intensity Factors Handbook," Vol. 1 ,Vol 2, Vol.3, Pergamon Press, 1987.

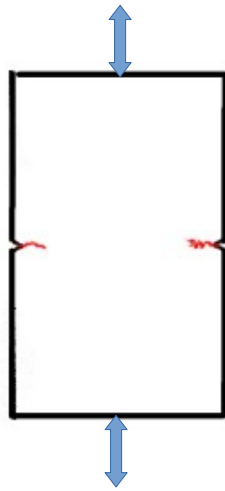
N.E. Dowling, "Mechanical Behavior of Materials, " 3<sup>rd</sup> Edition,

British Standard 7910, "Guide to methods for assessing the acceptability of flaws in metallic structures," ISBN 978 0 580 60108 8, 2005

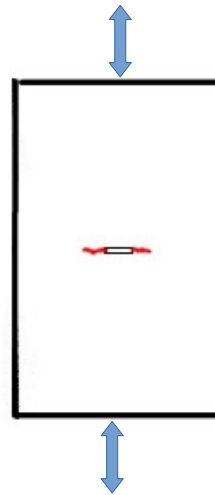
Constant amplitude specimen testing to create the basic material  $da/dN$  curves use a wide variety of geometries, but generally translate the test data into a  $\Delta K = K_{\max} - K_{\min}$ .



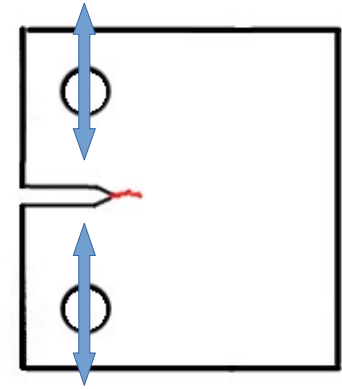
$$K = F_1 * Sg \sqrt{\pi a}$$



$$K = F_2 * Sg \sqrt{\pi a}$$



$$K = F_3 * Sg \sqrt{\pi a}$$

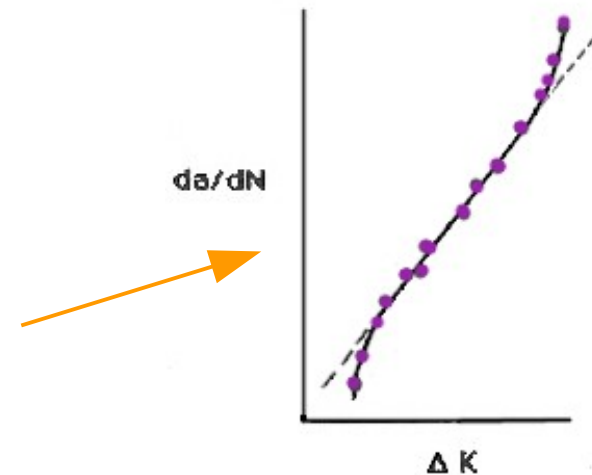


$$K = F_4 * Sg \sqrt{\pi a}$$

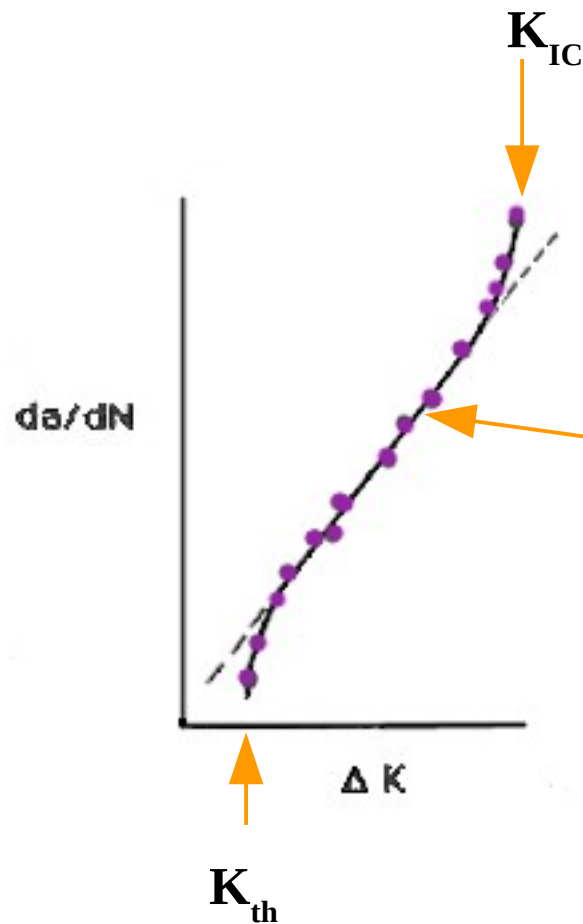
( For specimen testing standard see ASTM Std. E 647 )

Each specimen geometry requires a different computation of the correction factor  $F$

As a test proceeds and the crack becomes longer the user must record the crack length and the cycle number and then plot the data in  $da/dN$  (change of length per cycle) versus the  $\Delta K$  value at that crack length.



As the crack gets large the load in tension may cause the specimen to fracture. The stress intensity for this critical value is denoted as  $K_{IC}$



The middle of the curve can often be modelled with a straight line which, on a log-log plot suggests an equation of the form:

$$da/dN = A * \Delta K^m$$

It is called a "Paris" equation after its inventor  
P.C. Paris

If no test data is available, several of the design codes, such as BS-7910 (Sec. 8), provide values for  $A$  and  $m$ . A number of handbook compilations of  $da/dN$  data are also available. e.g.

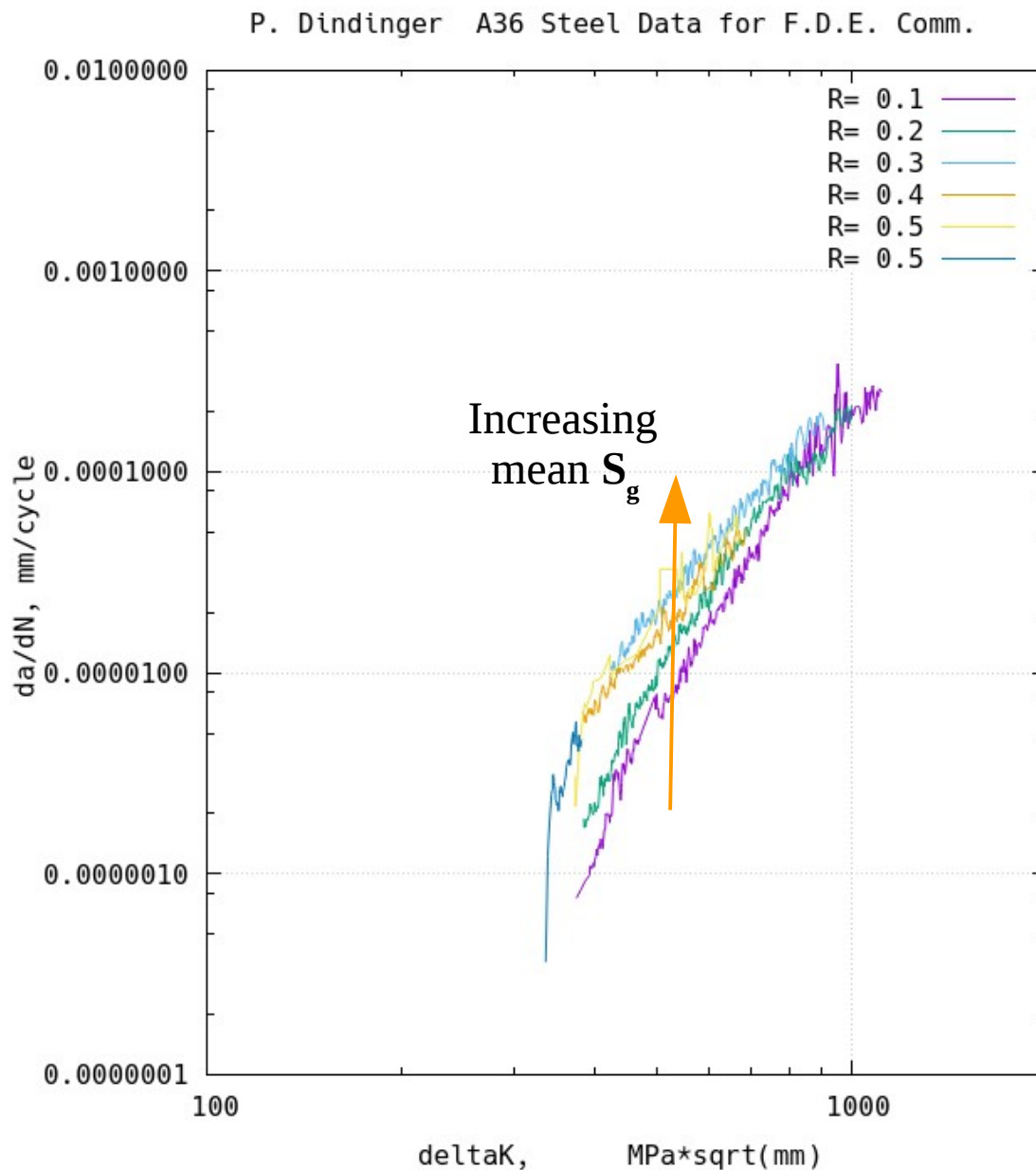
D.Taylor, "A Compendium of Fatigue Crack Growth Rates,"  
Engr. Advisory Services Ltd, UK, 1985  
also

<http://www.tc.faa.gov/its/worldpac/techrpt/ar05-15.pdf>

or

[http://everyspec.com/MIL-HDBK/MIL-HDBK-0001-0099/MIL\\_HDBK\\_5J\\_139/](http://everyspec.com/MIL-HDBK/MIL-HDBK-0001-0099/MIL_HDBK_5J_139/)

When the crack is growing very slowly per cycle it is assumed to be at a "threshold"  $K_{th}$



Mean stress effects are similar to those observed in crack initiation tests. With increasing tensile mean stress the  $da/dN$  curves move upwards and have a lower threshold  $K_{th}$

In the figure the test stress ratio  
 $R = S_{g \min} / S_{g \max}$

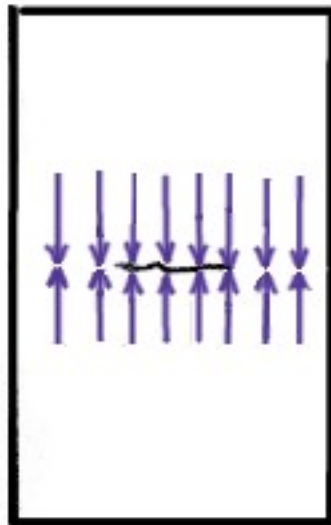
is a larger fraction as the mean stress increases.

Thus for a given  $\Delta K$  a test at higher mean stress will cause more crack propagation damage.

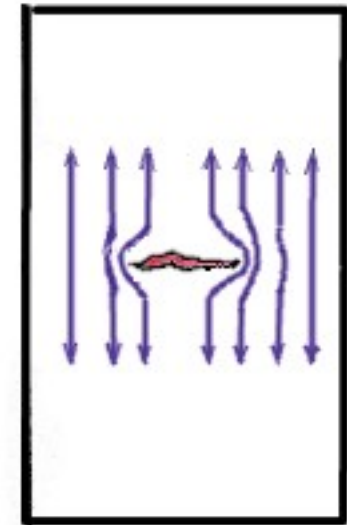
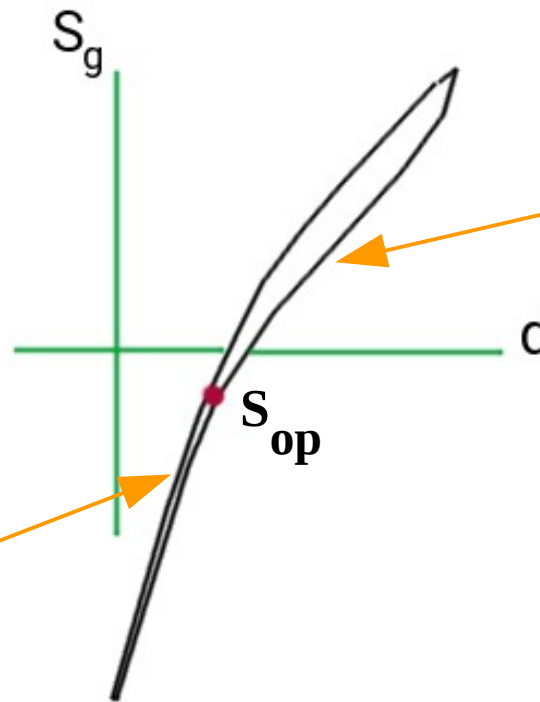
$$\text{damage} = da/dN$$

A variety of methods have been proposed to account for this "R ratio effect", much like the mean stress methods for crack initiation. These methods will not be covered here, with the exception explaining the effects of crack closure, a concept used by many of the methods.

When one examines a stress-deflection loop for a cracked specimen one sees a change in stiffness when crack is open



When closed the crack face transfers the stress.



When open the stress must flow around the crack and causes greater deformation in the uncracked section; plus much higher stresses at the tips of the crack.

Thus an effective stress intensity,  $K_{eff} = K_{max} - K_{op}$  can be computed and used to account for the effects of mean stress cycles

Plotting **da/dN** vs **ΔK<sub>eff</sub>** helps to explain the effects of mean stress (or R ratio) in crack propagation fatigue analysis, but observing the opening point in a test requires substantial experimental efforts.

As in the SWT mean stress correction parameter in crack initiation testing, a collapse of experiments run at different R ratios is also possible for crack propagation test results. There are a number of methods available, only one is presented here:  
Walker Equation:

$$\Delta K_{\text{equiv.}} = K_{\text{max}} * (1 - R)^y$$

For  $R < 0$   $y=0$  (when **K<sub>min</sub>** is in compression).

The exponent **y** is material dependent and can be found experimentally by iteration of its value and ensuring that the various R ratio test data for **da/dN** vs **ΔK<sub>equiv</sub>** collapses as close to one line as possible.

When computers are easily available one can also digitize the various R ratio test results and use interpolation to compute the effect of R onto **ΔK**. This latter approach is probably the simplest solution.

No single definitive method has been widely adopted. Thus in the next chapter on application we will simply use a **da/dN** vs **ΔK** visual best fit line to show how simulations can be performed. The software is "open source" and the user is welcome to make adjustments to reflect their own preferences.