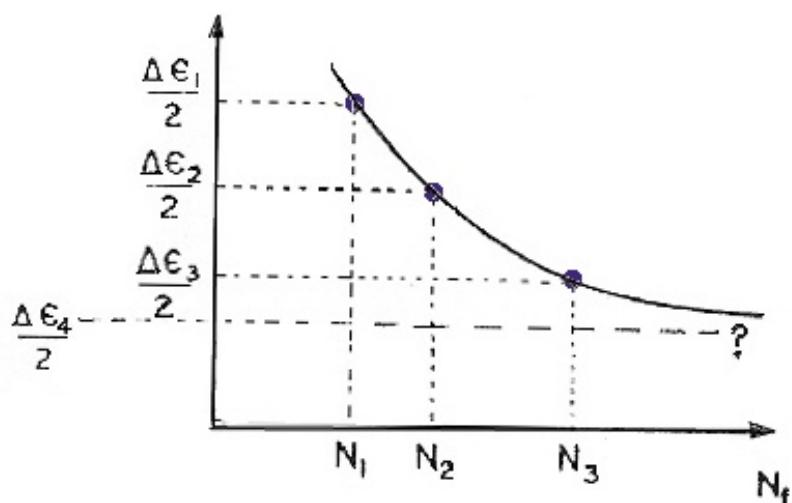
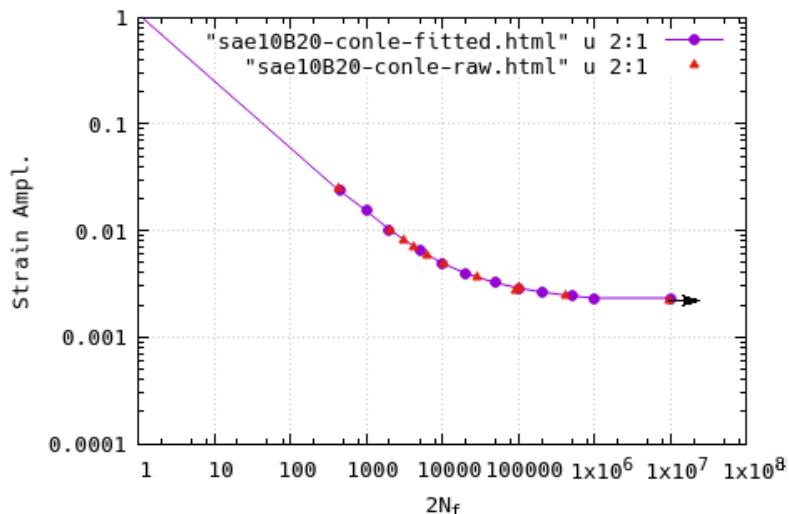


Damage below the fatigue limit



$$\text{Damage} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \dots$$

$$\text{Failure} \quad \sum \frac{n_i}{N_i} = 1$$

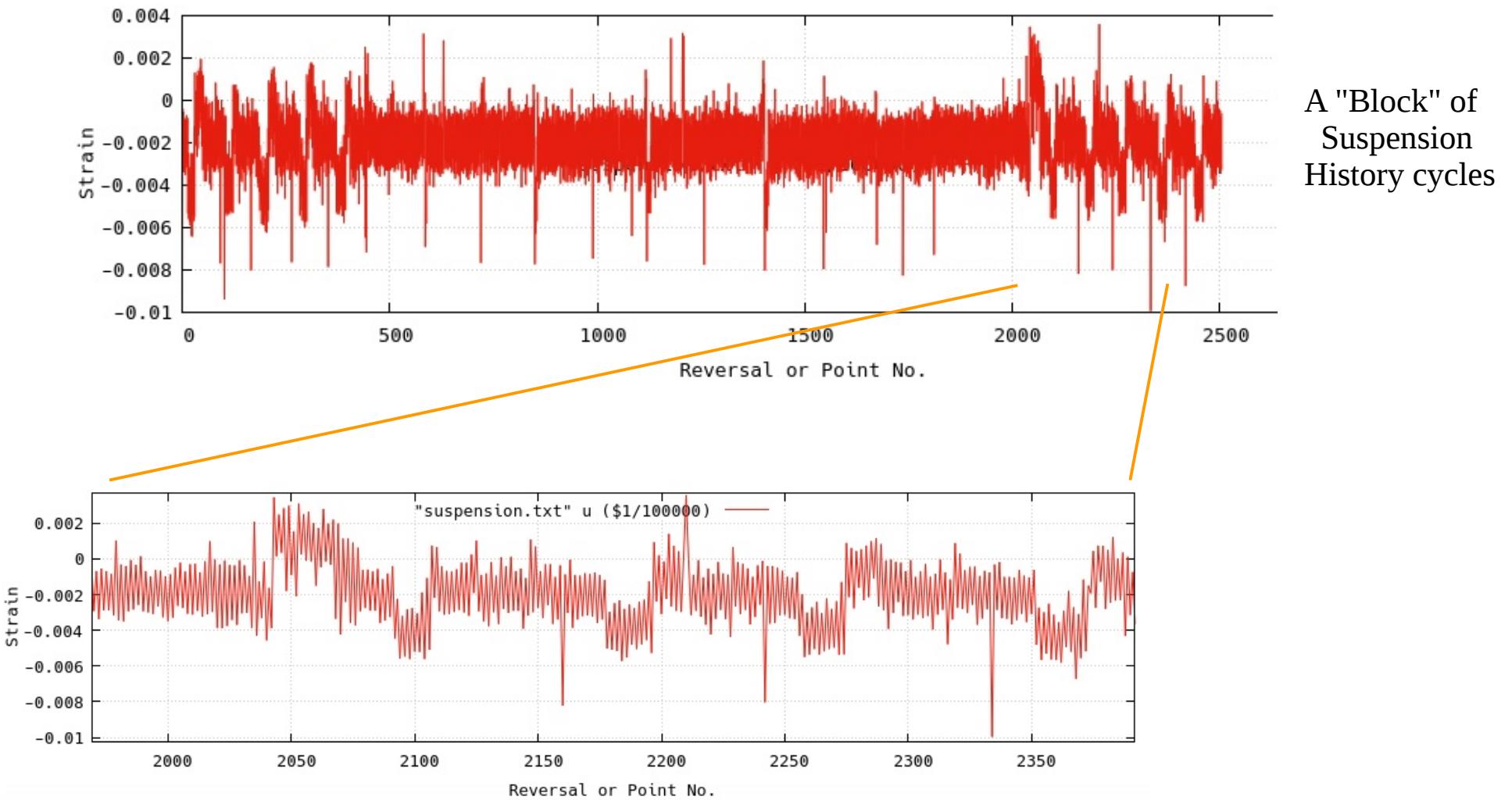
During constant amplitude (CA) testing, one generally starts with the short life specimens first and then tests the rest with the lower strains and longer lives.

In many materials at very long lives the curve will flatten out, indicating a "Fatigue Limit" strain or stress.

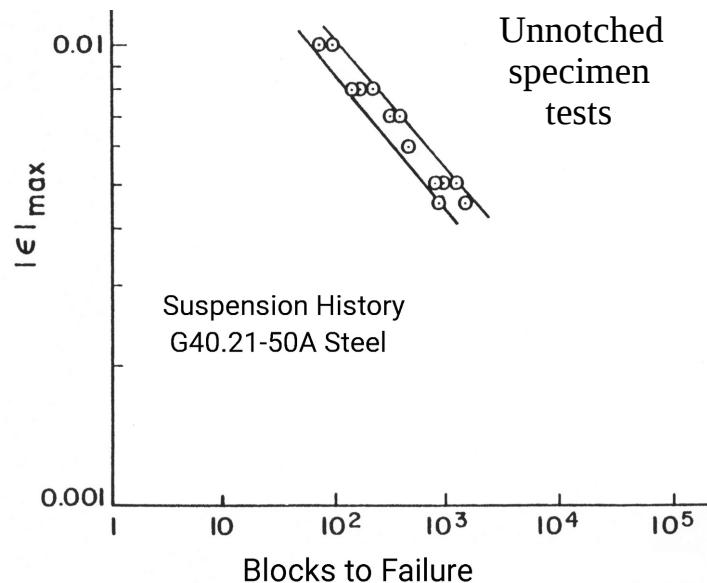
Thus when one encounters a resolved hysteresis loop whose strain amplitude (or mean stress parameter etc.) is below the fatigue limit, one would expect that the loop does no fatigue damage.

Reality in variable amplitude service histories is a little different.

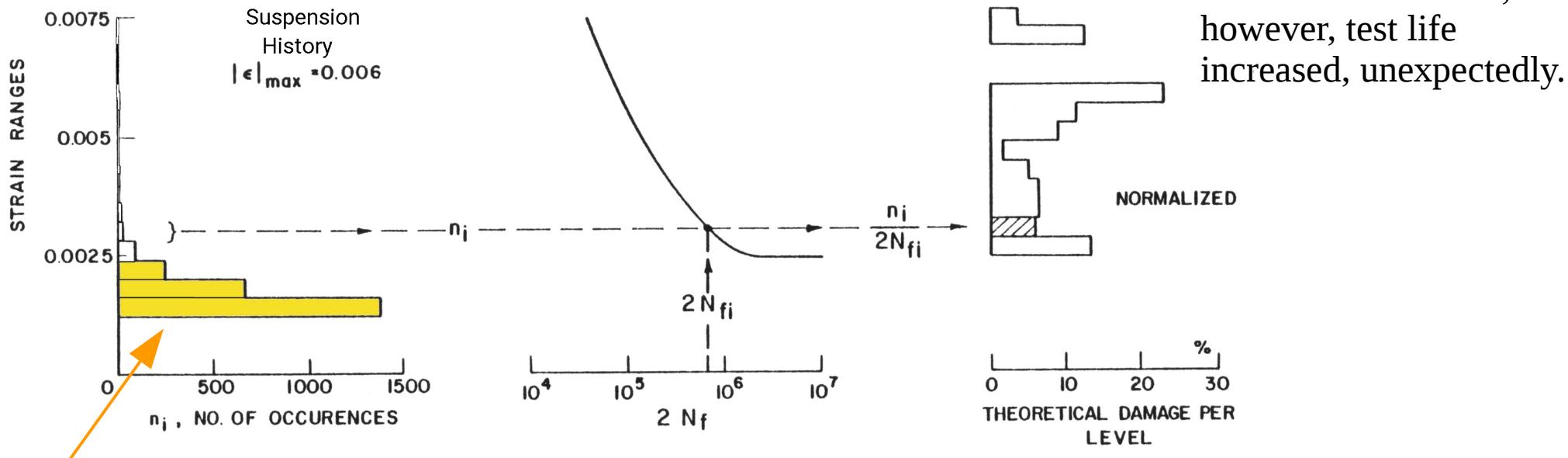
Many variable amplitude service histories contain lots of small cycles that are below the constant amplitude fatigue limit for a given hot-spot.



The history shown here was applied in a number of scaled down versions onto unnotched axial test samples



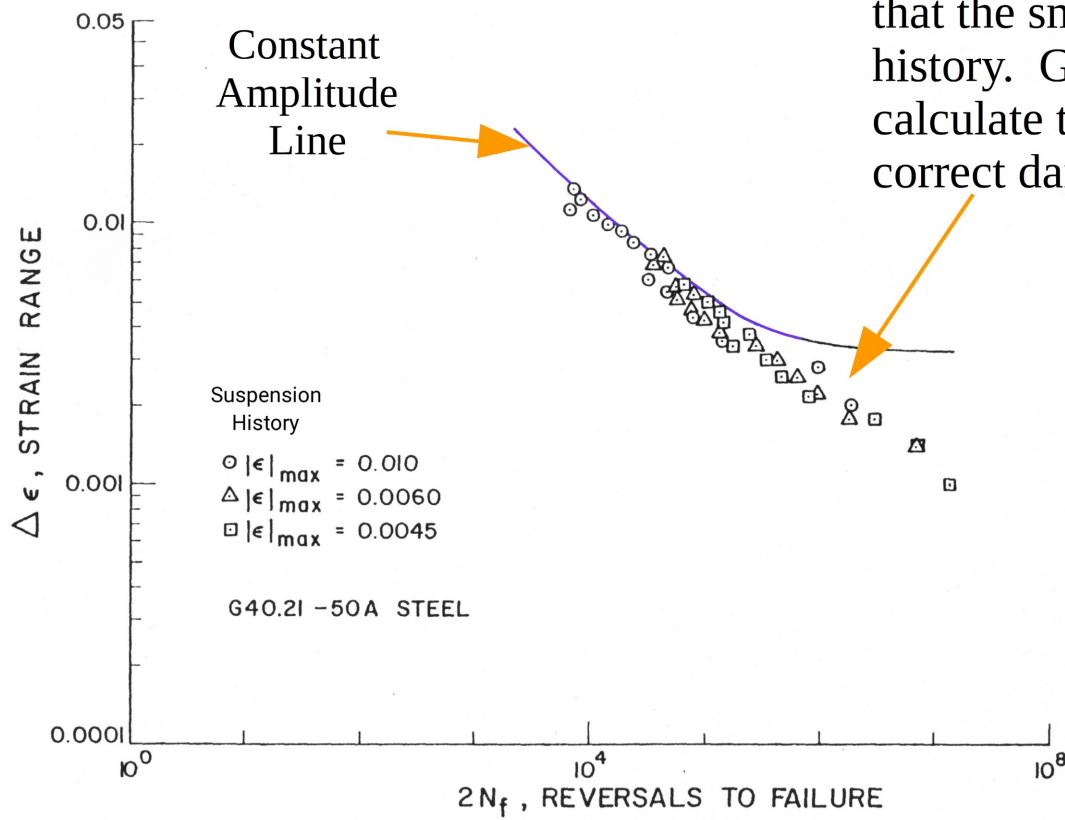
The Palmgren-Miner linear damage summation method for resolved closed hysteresis loops is shown in the lower figure. It predicts that a lot of damage is done by a very few large loops and that a large number of loops were below the fatigue limit.



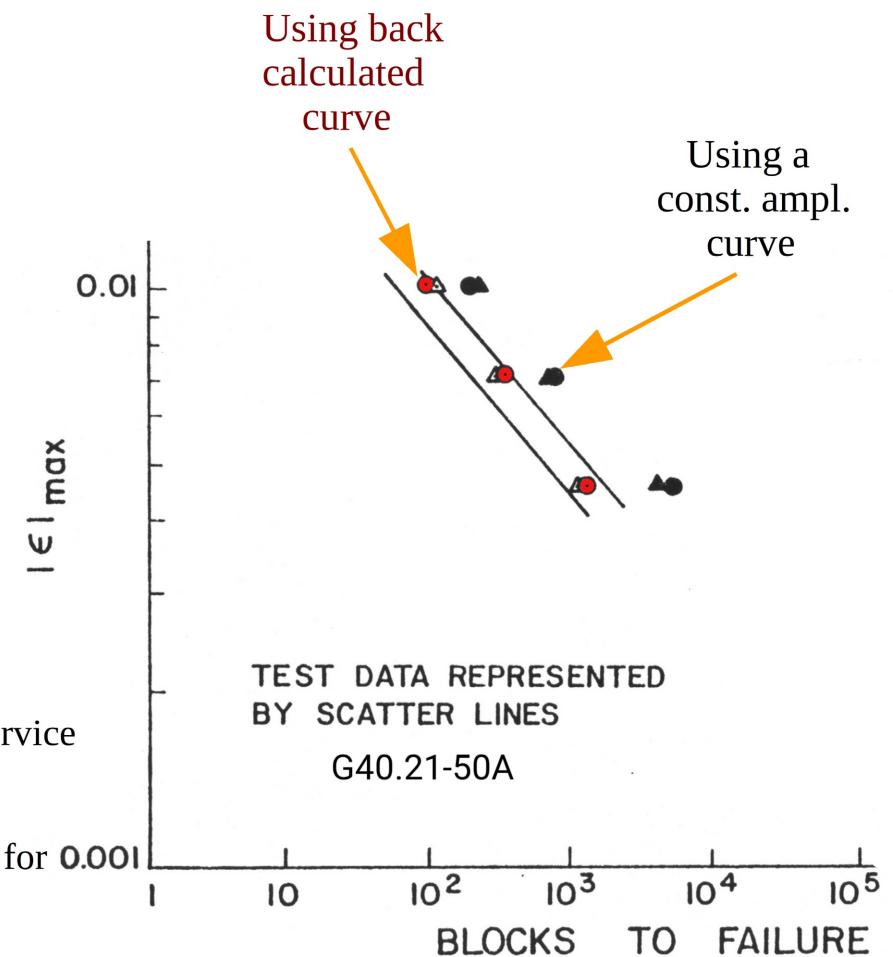
It was decided that for repeated tests one should be able to remove the loops below the fatigue limit and thereby shorten testing time.

When this was done, however, test life increased, unexpectedly.

No damage?



By removing small range cycles, starting with the smallest, and re-testing on a new specimen and observing an increase in life, we were able to compute the damage that the small range cycles caused in the original un-edited history. Given the damage contribution one can back calculate the actual strain life curve that would predict the correct damage summation.



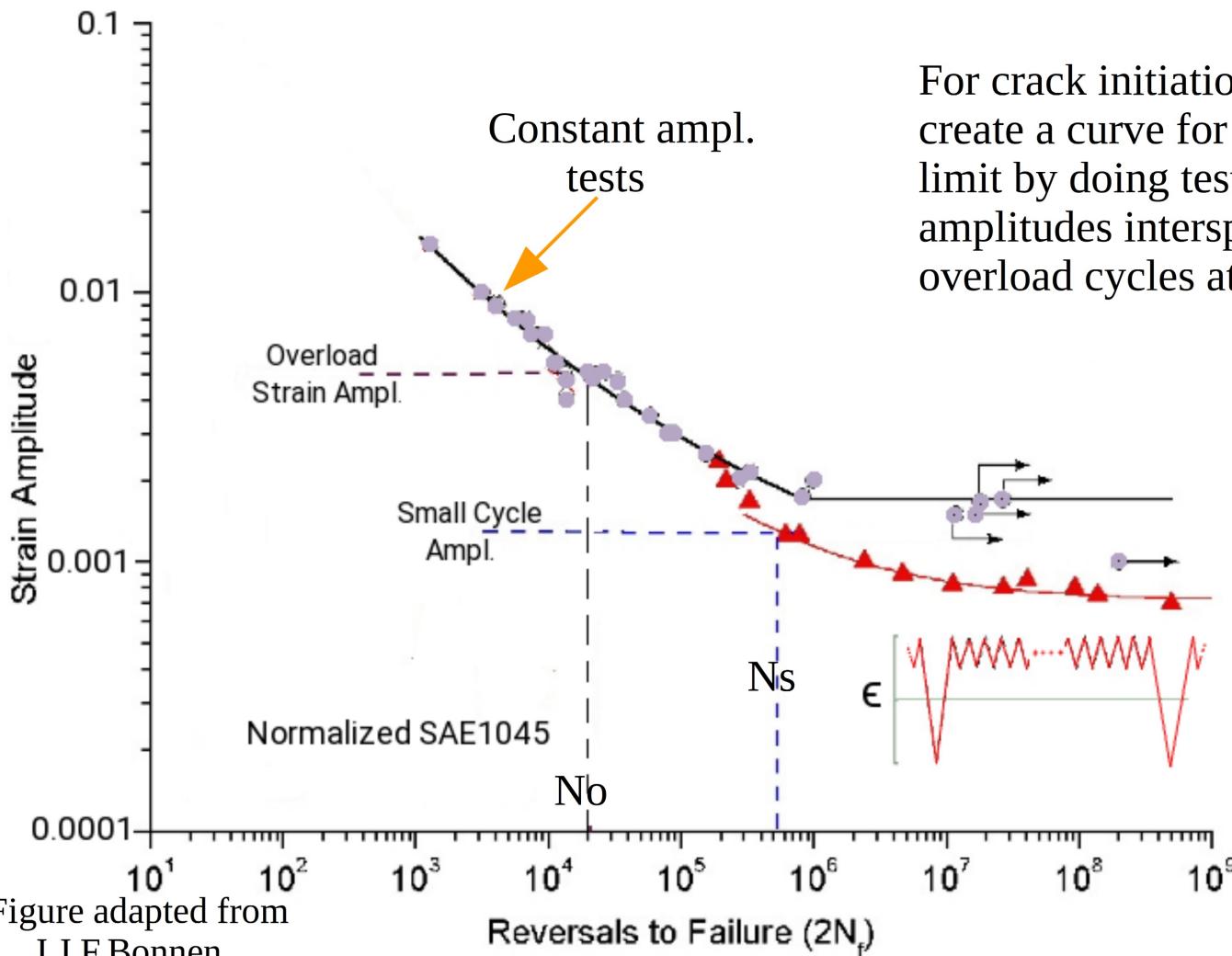
Refs.:

A.Conle, T.H.Topper, "Overstrain Effects during Variable Amplitude Service History Testing," Int. J. Fatigue, V2 N3 1980 p130-136

D.L.DuQuesnay, M.A.Pompetzki, T.H.Topper, "Fatigue Life Prediction for Variable Amplitude Strain Histories," SAE Paper 930400, Mar., 1993.

Research indicates that damage below the fatigue limit occurs when mixtures of large and small cycles occur in a history:

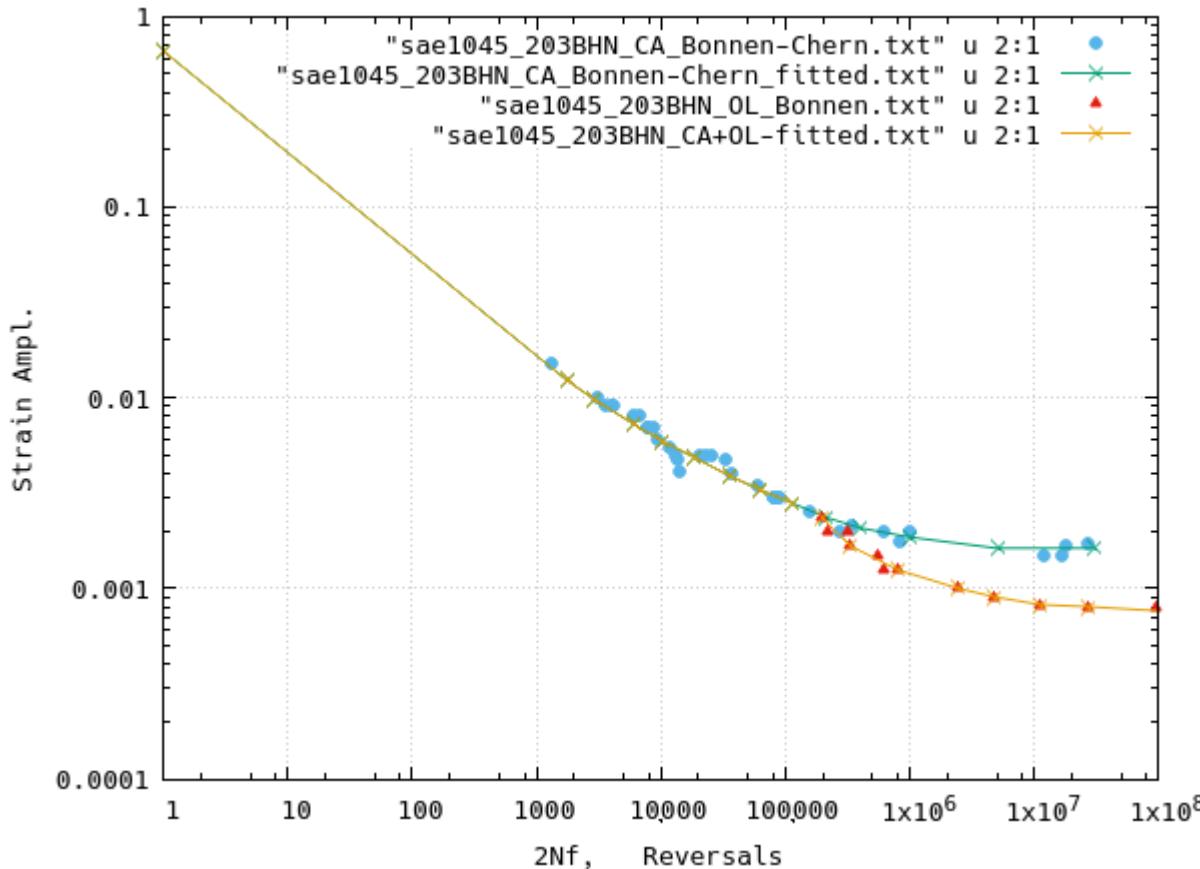
1. Large cycles can start a micro-crack which the small cycles can propagate
2. Constant amplitude cycles have a crack opening stress above the minimum stress. Large cycles, with their plasticity, lower the subsequent opening stress for the small cycles and make them more effective.



For crack initiation calculations one can create a curve for damage below the fatigue limit by doing tests with small cycle strain amplitudes interspersed at intervals with overload cycles at a large strain ampl.

When the test fails count the total number of big (No) and small (N_s) cycles and solve the equation for N_s

$$\frac{No}{No} + \frac{N_s}{N_s} = 1$$



The constant amplitude and overload raw and fitted points are shown in this plot.

For the POL fitted curve one starts with the CA fitted curve and adds in some of the raw POL points that are close to a good fit line. These points can then be tuned to a good fit by altering the life values.

As long as the stress-strain fit is maintained any life value can be given to a strain-life-stress triple point. A kink in the strain vs. life curve is no problem.

Exercise: Download the following files and plot them with gnuplot. One of the files contains the gnuplot commands.

1. <http://fde.uwaterloo.ca/FatigueClass/FCourseNotes/plotSAE1045-CA+OL.txt>
2. http://fde.uwaterloo.ca/FatigueClass/FCourseNotes/sae1045_203BHN_CA_Bonnen-Chern.txt
3. http://fde.uwaterloo.ca/FatigueClass/FCourseNotes/sae1045_203BHN_CA_Bonnen-Chern_fitted.txt
4. http://fde.uwaterloo.ca/FatigueClass/FCourseNotes/sae1045_203BHN_DL_Bonnen.txt
5. http://fde.uwaterloo.ca/FatigueClass/FCourseNotes/sae1045_203BHN_DL_Bonnen_fitted.txt
6. http://fde.uwaterloo.ca/FatigueClass/FCourseNotes/sae1045_203BHN-CA+OL.txt
7. http://fde.uwaterloo.ca/FatigueClass/FCourseNotes/sae1045_203BHN_CA+OL-fitted.txt

Whether to apply, or not to apply, this below the fatigue limit curve to your fatigue crack initiation damage prediction method calls for your engineering judgment.

The "periodic large cycles" must cause some plasticity and occur often enough to activate the effect.

If in doubt perform a separate prediction for constant amplitude (CA) curve and another using the periodic overload (POL) curve.

We will be considering these methods again in the chapters on applications.

Ref. for POL tests:

J.J.F. Bonnen, PhD thesis, Civil Engr., U.Waterloo 1998.

Note! Please also see Addendum on next page.

Addendum Jan. 2021:

I found a paper by M.Kikukawa, M.Jono and J.Song which also documents fatigue damage below the regular constant amplitude fatigue limit. Note that it was published in **1972**, a considerable number of years before the publications mentioned in the previous pages.

The paper can be obtained at:

https://www.jstage.jst.go.jp/article/jsms1963/21/227/21_227_753/_pdf/-char/en

The formal reference in Jstage is:

Makoto KIKUKAWA, Masahiro JONO, Jiho SONG,
“The Cyclic Plastic Strain and Cumulative Fatigue Damage
Fatigue Damage Caused by the Stress Below the Fatigue Limit”
J. of the Society of Materials Science, Japan
1972 Volume 21 Issue 227 753-758
Published: August 15, 1972
Released: June 03, 2009
DOI <https://doi.org/10.2472/jsms.21.753>

Due to copyright I cannot depict the critical figure 9 here. The paper text is in Japanese but the Figure captions etc are in English and quite clearly show that fatigue damage occurs below the constant amplitude fatigue limit.

F.A.C 2021