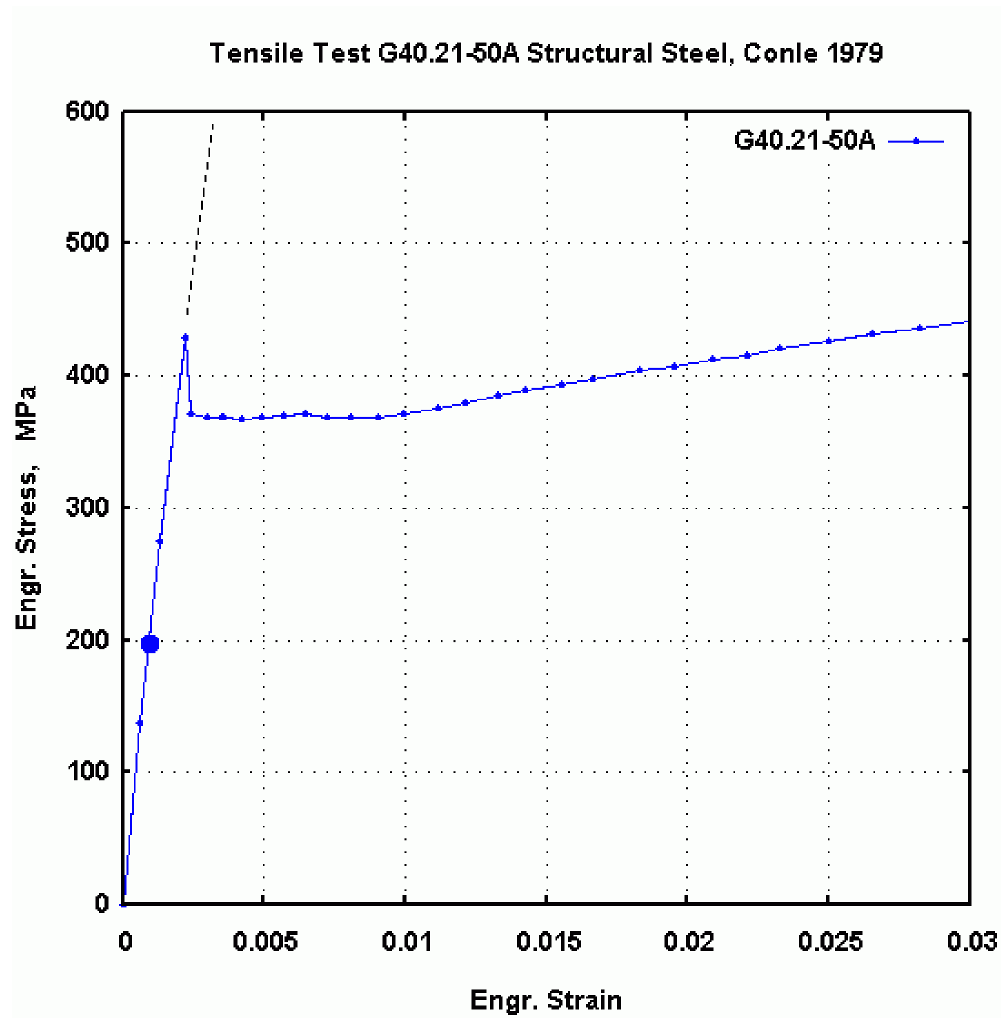
Chapter 7

Notches, Stress-Raisers and Finite Element Analysis

In much of structural design, such as building beams or columns, the stresses are kept well below the yield stress in order to allow for unexpected small overloads etc.

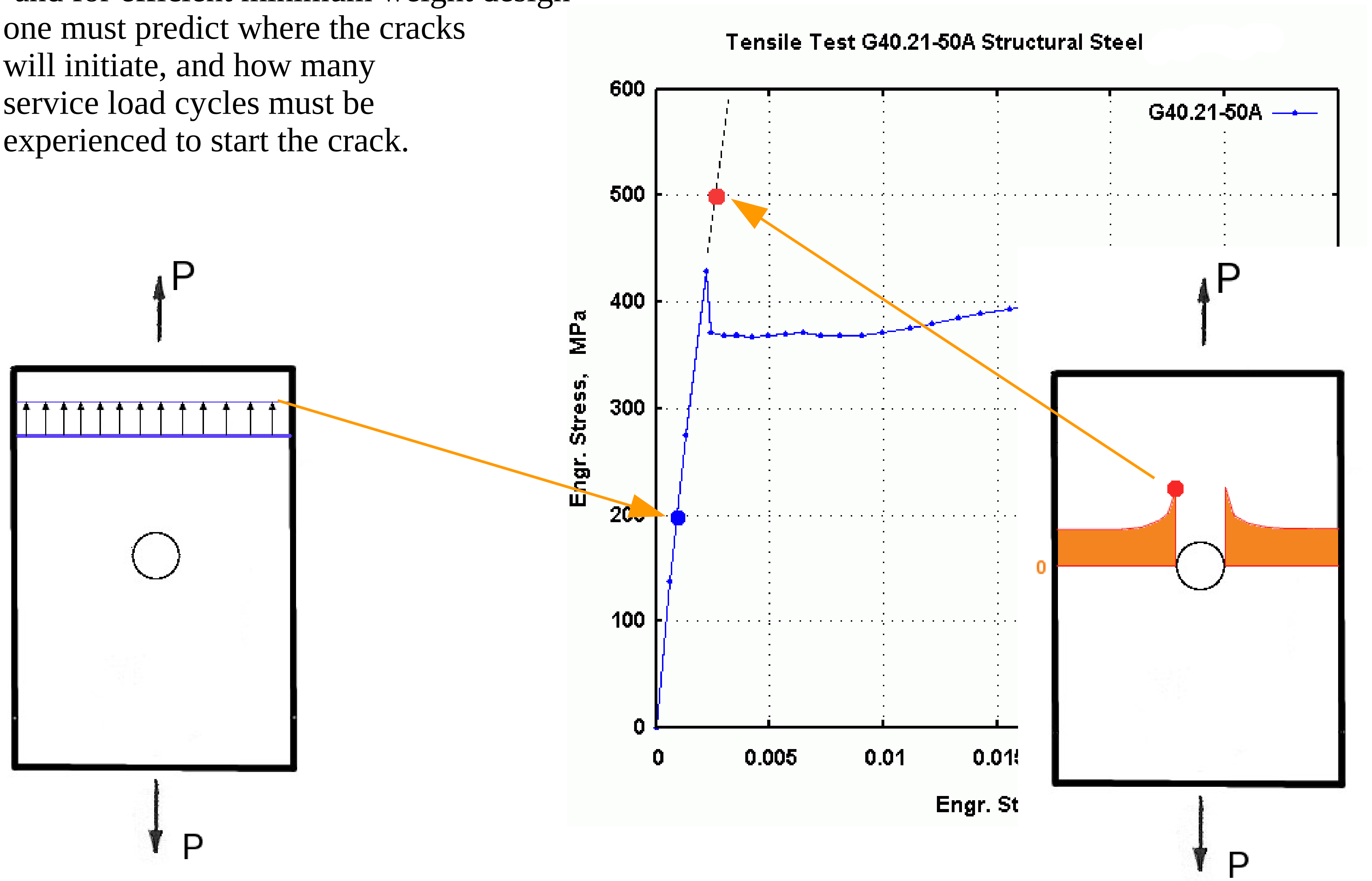
A customer would not appreciate, for example, if while driving over a pot-hole, the car would take on a permanent bend due to plasticity.



At bolt holes, or welds, or other geometric discontinuities, some plastic (over the yield) regions are allowed, as long as these small zones are constrained by the larger structure's elastic field.

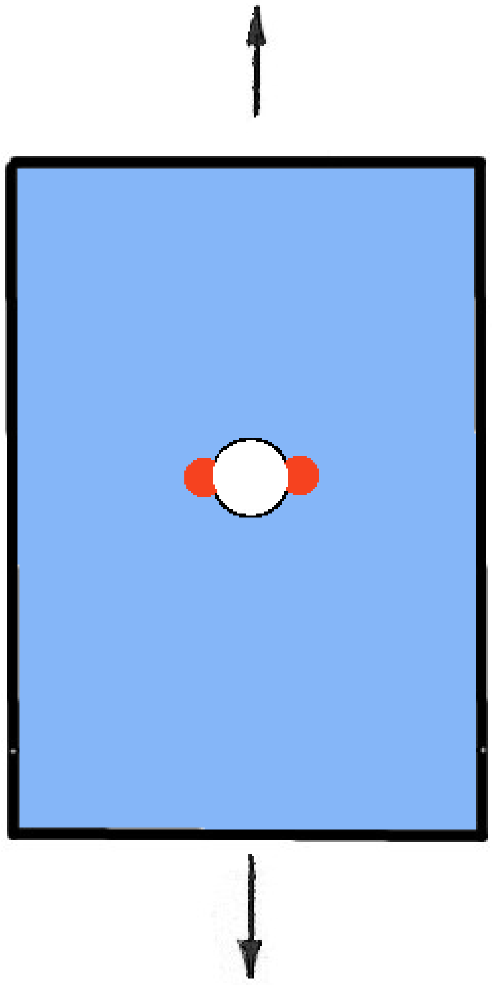
Fatigue cracks will start in such small plastic zones.
Stress raisers are unavoidable.

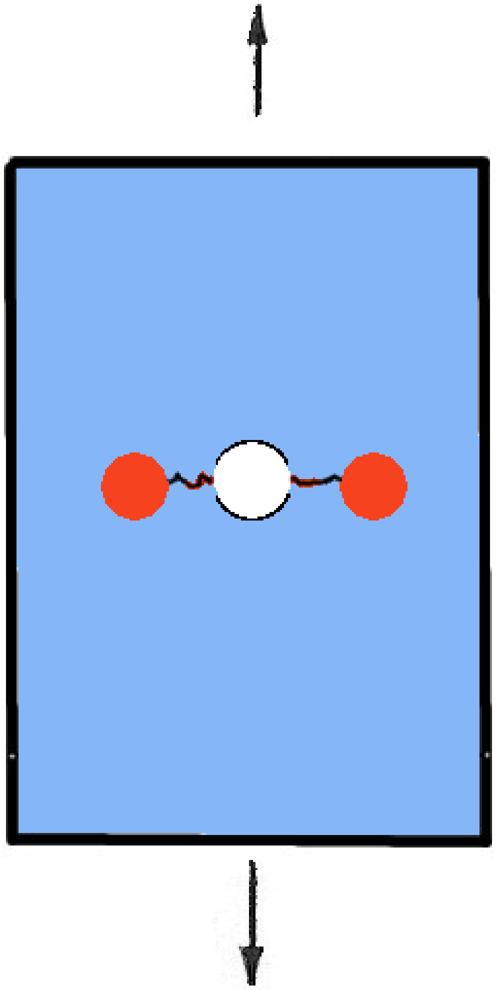
-and for efficient minimum weight design one must predict where the cracks will initiate, and how many service load cycles must be experienced to start the crack.



A small plastic zone (red) will not hurt a component that is mostly elastic(blue). The zone is con-strained. The trade-off is that you may get a fatigue crack starting. Hence the need for a fatigue analysis to predict life..

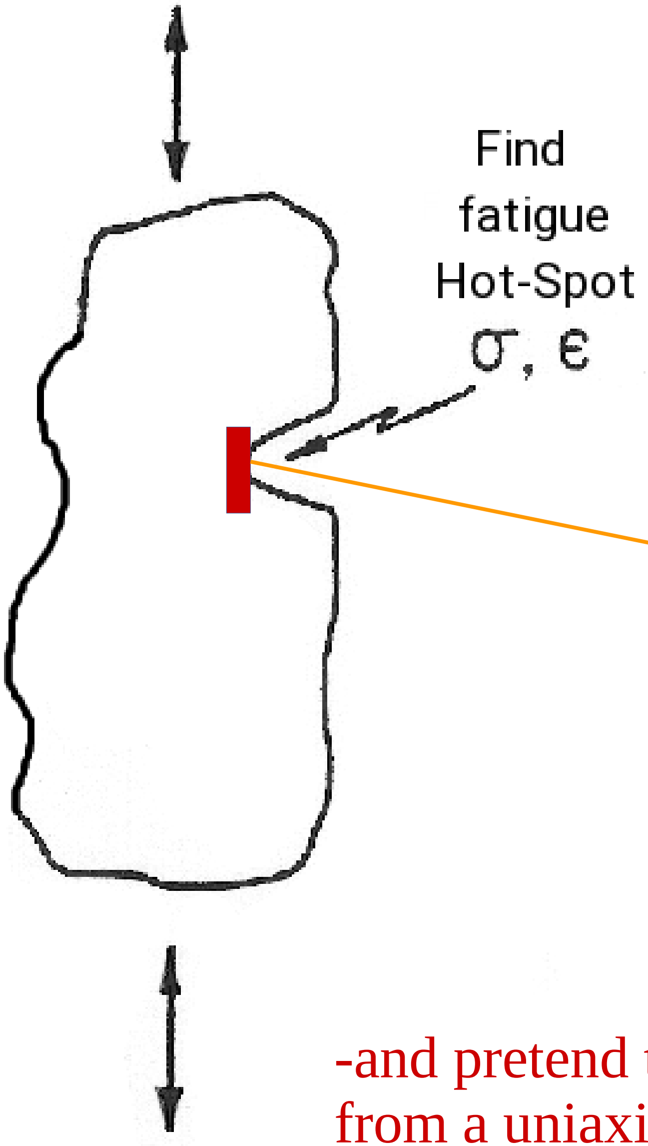
As the crack propagates, the plastic zone gets bigger, if +/-Sgross remains constant. At some point the net section will go fully plastic and the component will distort and become unusable.

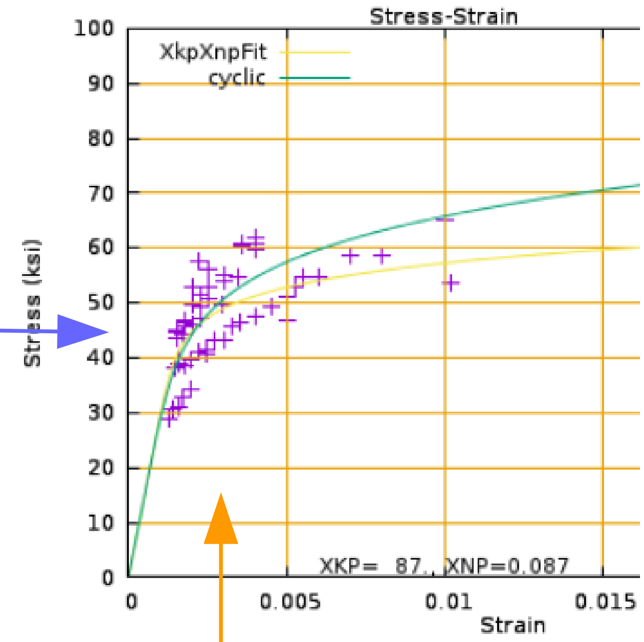




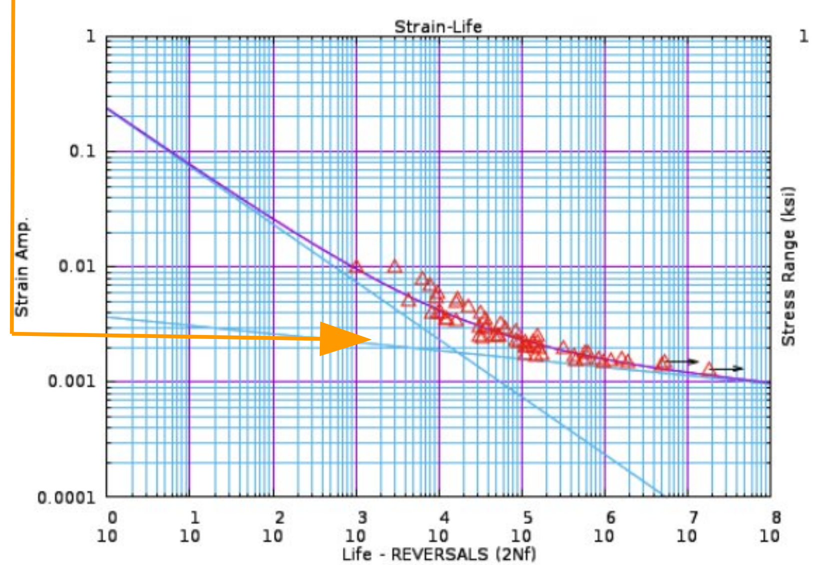
The objective of a notch analysis:

Find the hot-spot stress and strain



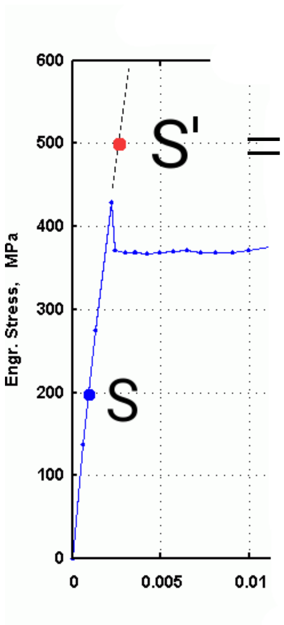


Then compute expected life.



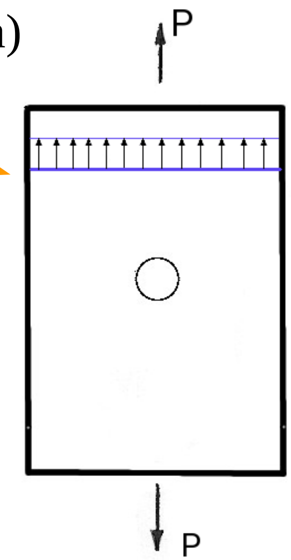
-and pretend that they are from a uniaxial test sample

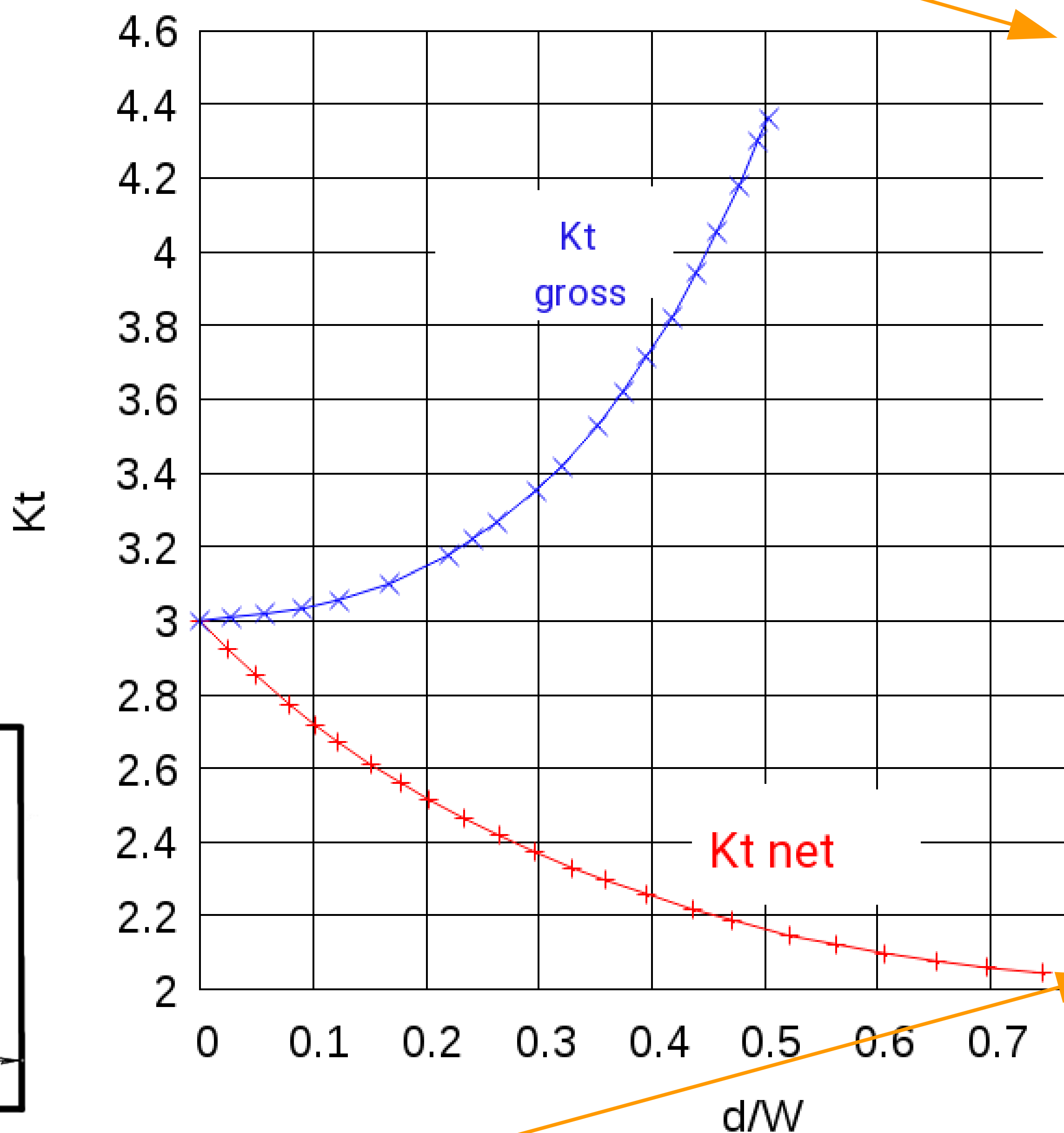
Kt : Elastic Stress Concentration Factor

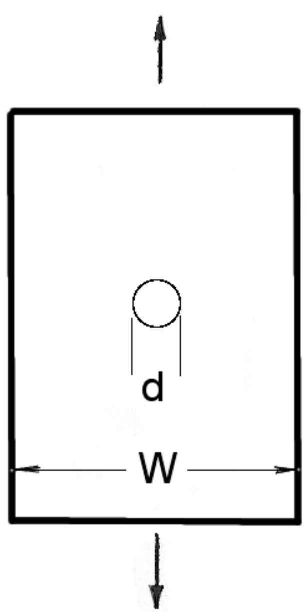


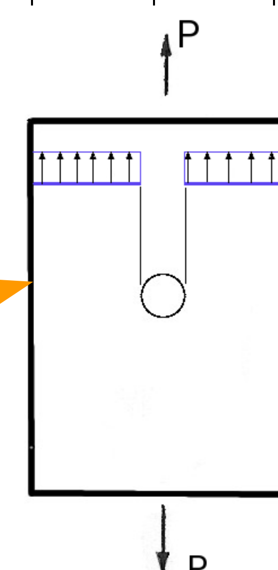
$$S' = Kt * S$$

S' can be either "Gross" stress = P/(total Area)









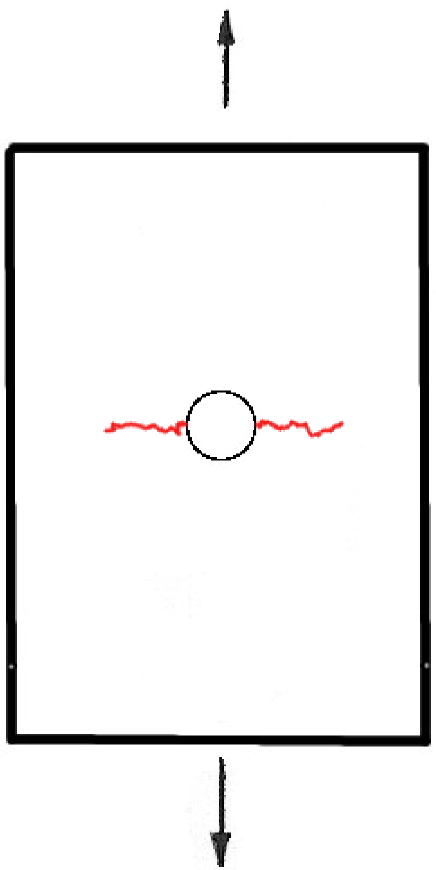
or "Net" stress= P/(Area without the hole)

"Area" refers to cross-section area

Different industries or prediction methods use either Snet or Sgross.

Snet is generally used for crack initiation fatigue life predictions.

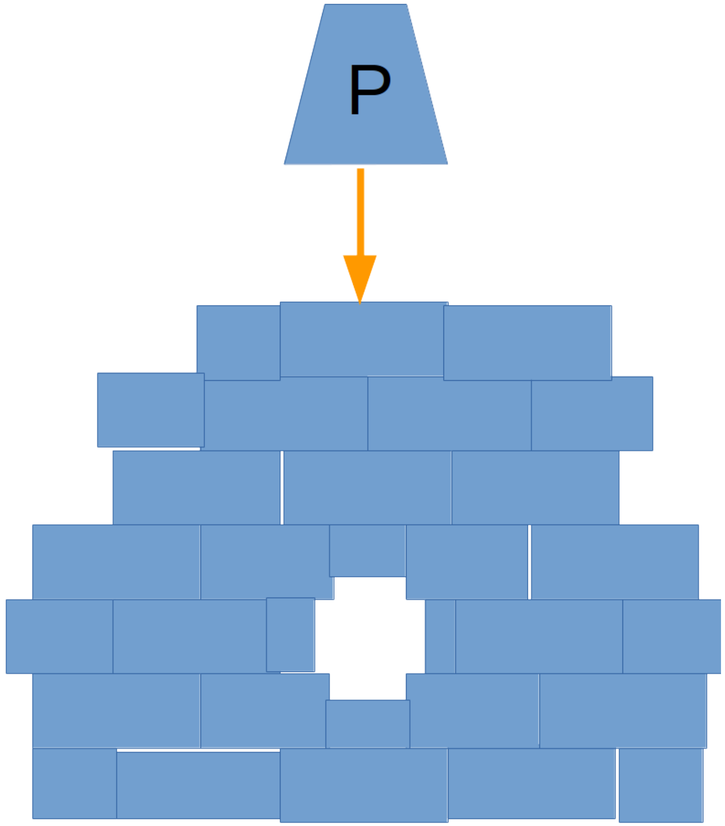
Sgross is used for crack propagation life prediction.
As the crack grows it changes the net area, thus using Snet would require constant changes. Using Sgross avoids this complexity somewhat.



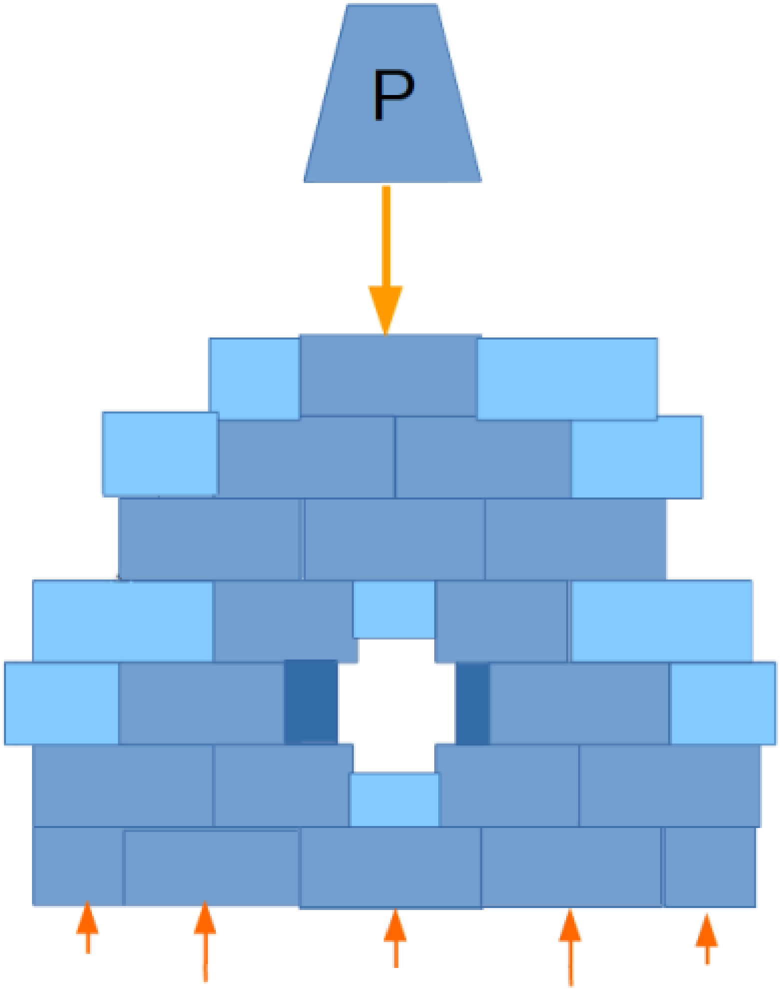
Hot-Spot stress determination using Elastic Finite Element Analysis (FEA)

(If you are already familiar with FEA you can skip these pages)

FEA represents a component or structure as a collection of "building blocks" They can have various shapes and sizes. In the rough example here we are using a bunch of elastic (rubbery) bricks that are glued to each other on all contacting sides.



When the weight P is placed on top of the bricks the whole structure will deform and each brick will deform according to the amount of weight it passes through itself.



Each rubbery brick is spring-like. i.e.: elastic

The structure and the bricks must obey the rules of

Compatibility: what was glued together must remain glued together

and

Equilibrium: All the external forces must add up to zero and the forces on each brick must add to zero.

FEA solves this numerical problem.
A balance between
compatibility and equilibrium.

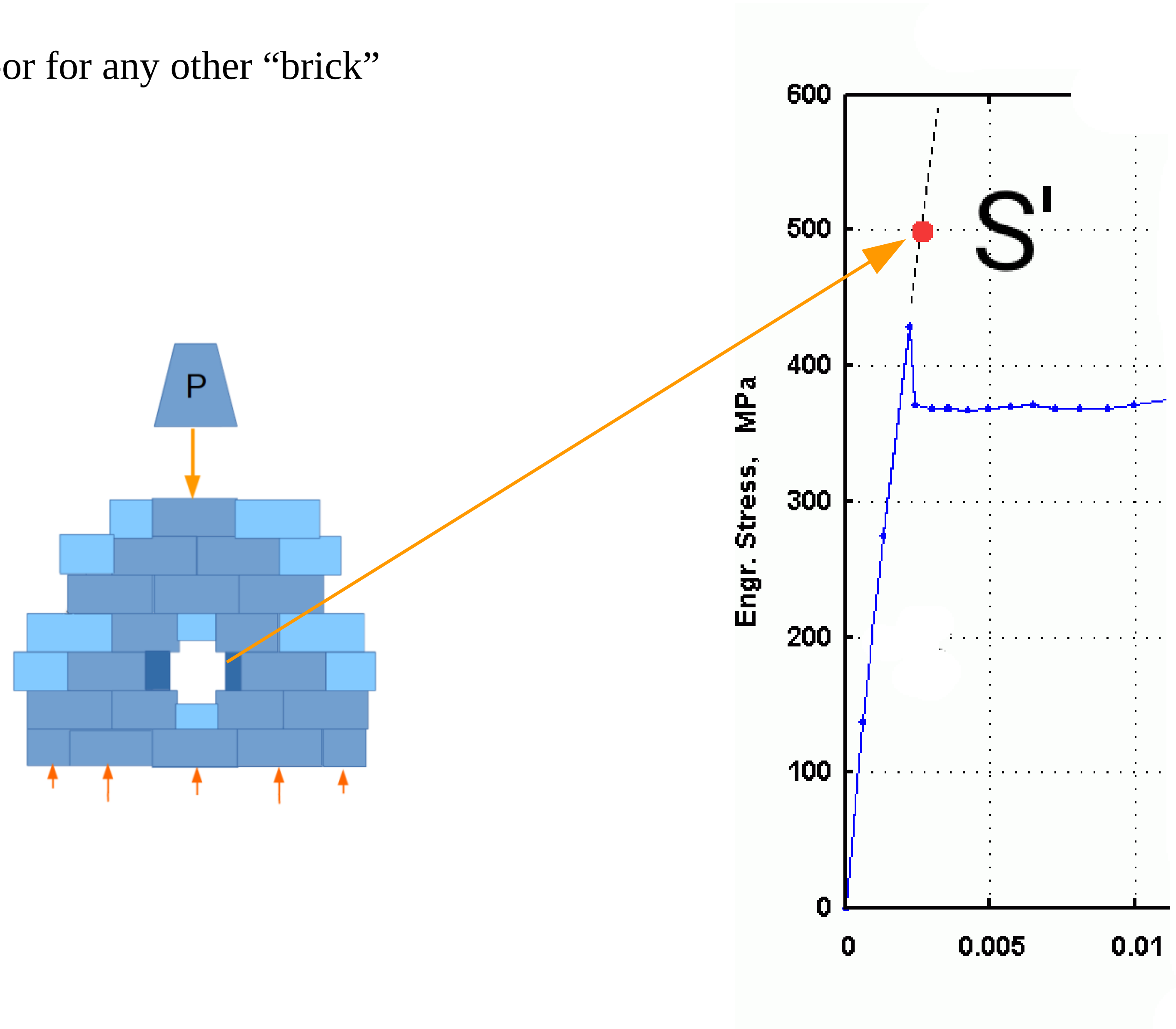
The forces on each brick are translated into elastic Stresses.

In the rough analog above the darker shaded bricks carry more load or stress.

Ok, ok it looks like an Inuksuk Pure coincidence. Well… this course is a Canadian project :-)

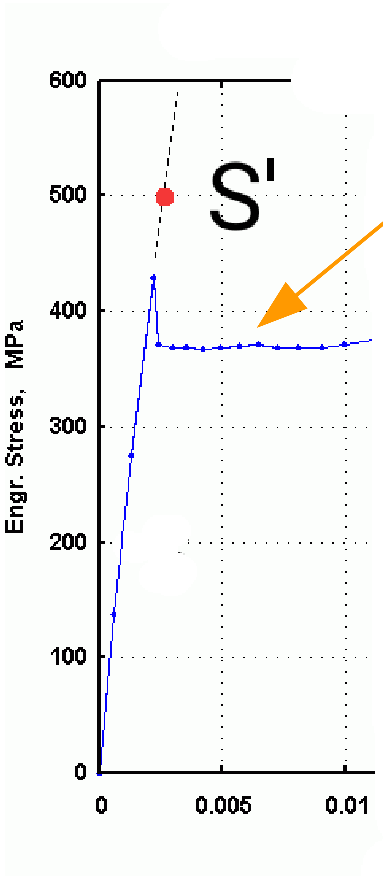
Like the Kt analysis, Elastic FEA will give us the elastic stress expected at a hot-spot.

-or for any other "brick"



“**But**,”   you might ask, as many engineers do, “**this stress is above the Yield Stress!?**”

The material would flow plastically before it could ever get to this stress.



Yes, we need a Plasticity Correction Method to approximate a more realistic stress and strain.

There are a few methods for converting the elastic stress results from a Kt or FEA analysis to correct for plasticity.

The most popular at the moment is the "Neuber Plasticity Correction"

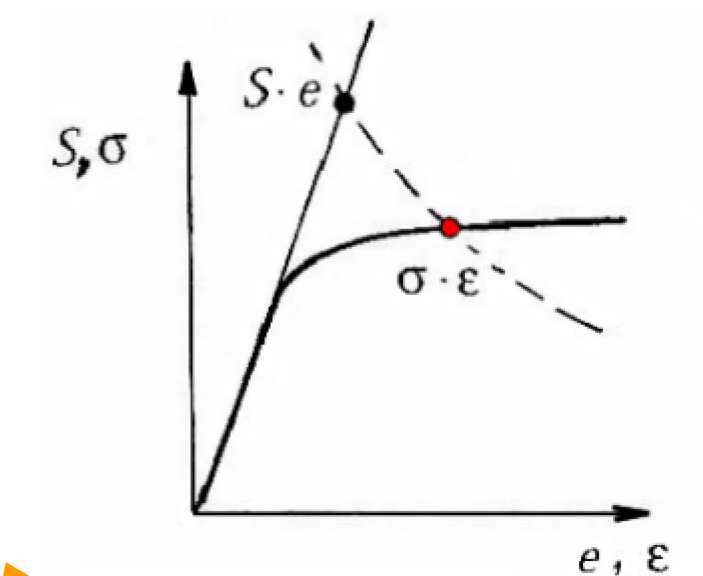
Ref.1 : H.Neuber, "Theory of Stress Concentration for Shear Strained Prismatic Bodies with Arbitrary Non-Linear Stress-Strain Law," J.Appl.Mech, Trans. ASME V28 N4, Dec. 1961 pp.544-560

Ref.2 : T.H.Topper, R.M.Wetzel, J.Morrow, "Neuber's Rule Applied to Fatigue of Notched Specimens," ASTM, J.of Materials V4 N1, March 1969, pp.200-209.

Given that

Stress x Strain = Energy

one can compute the equivalent stress and strain on the plastic deformation curve by setting the elastic analysis energy to the plastic curve energy



$$\sigma \cdot \varepsilon = S \cdot e$$

It can be solved by stepping along the "real" stress-strain curve in small increments, computing the energy at each step, and comparing the result to the elastic energy.